Energy vs. Momentum

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1 Introduction

Most documentation on bullet calibers and loads for self-defense emphasize bullet energy; comparisons of "stopping power" between calibers are also normally done in terms of energy. Here, we examine why muzzle energy is a mostly useless number for studying the effectiveness of bullets and calibers. For the rest of this document, all comparisons will be done assuming that a given person can shoot all calibers equally accurately; shot placement is more important than any other factor, but is beyond the scope of this document.

To start, we will look at what is kinetic energy, then explore why this is not a useful concept for bullet effectiveness, and end with why momentum offers a better measuring stick for calibers and loads.

2 What is energy?

Energy is a measure of the amount of work that can be done by a system. In the case of a fired bullet, the energy of interest is purely kinetic, and will eventually be dissipated as strain (deformation of a target and bullet) and heat. Kinetic energy is defined as:

$$K = \frac{1}{2}mv^2$$

The dominant term in this equation is the velocity (v). This means that comparisons between calibers based on energy are comparisons of velocity.

3 Why doesn't energy work?

The problem occurs in that energy, while a useful notion in physics, is not particularly effective in determining the lethality of a bullet. This is because of the nature of terminal ballistics - the behavior of a bullet when it enters a target, and the mechanics of wounding. [The following points are taken from the U.S. Department of Justice reports on handgun wounding and caliber selection for law enforcement agencies; both are available on the WWW as PDF files.]

The only two factors of importance when looking at effectiveness (lethality) of a cartidge are:

1. Penetration - the bullet must penetrate far enough into the target to reach vital organs or the central nervous system. Note that as long as a bullet can reach vital organs, additional penetration is irrelevant.

2. Size of hole - the larger the permanent hole generated in the target, the faster it bleeds to death, or the less precise the shot needs to be to destroy the central nervous system.

[Rifle rounds, due to their high velocities, produce trauma from temporary cavities which are much larger than the bullet diameter. This affects the precision to cause a lethal hit, and increases the bleed-out rate. Hence, rifles are more lethal for a given bullet diameter. This is only an important point when comparing handguns against rifles.]

Energy does not accurately correlate with either of these factors. Penetration is a linear function of velocity and cross-sectional density of the bullet. The higher the cross-sectional density and velocity, the greater the penetration. However, cross-sectional density is a function of bullet geometry and mass; for similar geometries, density is purely dependant on bullet mass. Size of the permanent cavity is dependant on the diameter of the bullet and the degree of expansion or fragmentation when the bullet travels in the target. For both lethality factors, scaling is linear with velocity and mass, not quadratic, which is why energy does not correlate well.

Small, fast bullets are useful for military applications, where the purpose is to wound, not kill, human targets that may have body armor; the speed and high cross-sectional density of the light bullets gives them deep penetration, but the small diameter yields lower lethality.

4 Momentum, a better measure

Momentum is defined as:

$$p = mv$$

Note that momentum is a linear function of both velocity and mass. This makes momentum a better theoretical correlation with lethality. Momentum is not the perfect lethality measure, as there is no dependance on bullet geometry, nor is there a relation for the uselessness of over-penetration.

Since ammunition manufacturers and hand-loaders routinely generate muzzle velocity data for computing muzzle energy, the computation of momentum is trivial. Given the ease of computing momentum, and the better theoretical correlation, more investigation into using momentum as a lethality index is definitely warranted.

5 Momentum of common rifle and pistol cartridges

Below are two graphs of momentum vs. distance, one for common rifle cartridges, and one for common pistol loads. All data were taken from the ballistics tables available from Remington. Hence, the numbers represent ballistics for Remington, UMC, and associated companies loads. Individual handloads, or even other factory loads, may vary significantly from these values; computation of momentum is easily done if the bullet velocity is known.

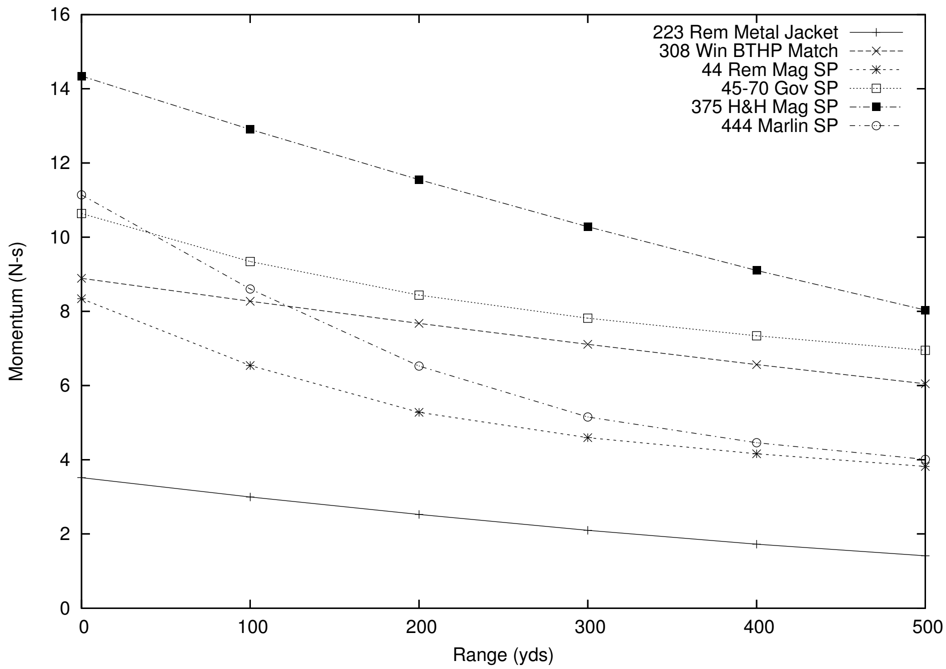

Figure 1: Momentum vs. range for common rifle cartridges. Momentum is in SI (metric) units; N-s = kg-m/s = 0.2248 lb-s.

Note that the rifle plot shows no surprising results. Large, fast cartridges retain high momentum from the muzzle to long range (e.g. .375 H&H Magnum), while large, slow calibers show high momentum that rapidly diminshes with distance (e.g. .45-70 Gov't). From the graph, it is clear that the .223 Remington is a vastly less lethal caliber than .308 Winchester, or any of the large-bore rifles. This has plenty of anecdotal support from hunting and combat reports. The graph also show why .45-70 and .444 Marlin are considered short-range hunting cartridges; momentum is initially quite high, but rapidly falls with distance to almost 1/3 the original value.

Suitably shaped bullets, such as the Spitzer (pointed) bullets of the .223 Remington, .308 Winchester, and .375 H&H Magnum, retain far more velocity, and hence momentum, at longer ranges. This makes them far better long-range hunting cartridges than the large-bore flat nose cartridges (.44 Remington Magnum and up). However, at the highest velocities (.375 H&H), even pointed bullets cannot prevent large velocity drop over 500 yards. Considering the very high muzzle velocities, the momentum at 500 yards is still quite formidable.

For rifle to rifle comparison, momentum appears to correlate well with anecdotal evidence of lethality, although a more thorough study and comparison would likely turn up deviations. In particular, a more complete study needs to account for bullet shape differences as they affect wound results in the target. For similarly shaped bullets, however, raw momentum comparisons appear to work remarkably well.

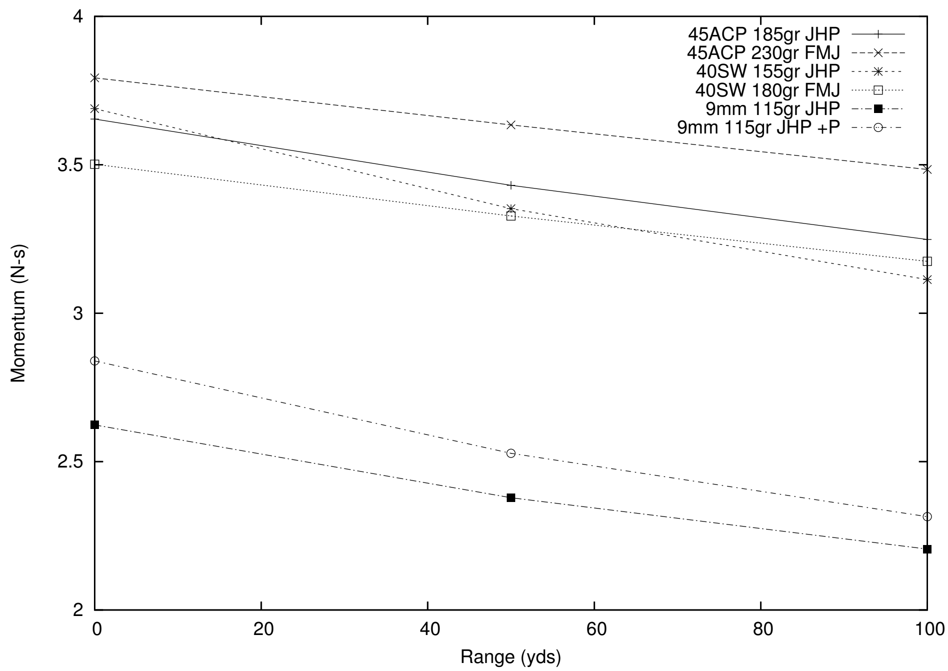

Figure 2: Momentum vs. range for common pistol cartridges. Momentum is in SI (metric) units; N-s = kg-m/s = 0.2248 lb-s.

Figure 2 plots momentum against range for three common pistol calibers and a variety of loads for each caliber. First, note that the plot range for momentum is equivalent to one interval on the rifle plot (figure 1), and hence the pistol cartridges are predicted to be far less lethal than all rifles but the .223 Remington. This is supported by anecdotal evidence (no hunters use these cartidges, etc.).

Second, note that the 9mm Luger 115 grain jacketed hollowpoint has the lowest momentum at all ranges, and hence is predicted to be the least lethal cartridge on the plot. Of the cartridges listed in the plot, this is not unexpected, as all other cartridges are moving faster (9mm Luger +P), or make significantly larger holes in the target (.40 S&W or .45 ACP). The wound size of an expanded 9mm hollowpoint may be larger than the wound size of the large-caliber metal jacket rounds (near-zero expansion), although the metal jacket rounds will penetrate much farther into a target than the 9mm hollowpoint.

If the relation between momentum and lethality always holds, then of the rounds plotted in figure 2, the most lethal round should be the 230 grain full metal jacket .45 ACP cartridge. Against human targets, this may not be the case; human targets are normally relatively soft (compared to bears, for example) and hence the full metal jacket bullets may cause significant overpenetration. This would reduce the lethality compared to the hollowpoint bullets, which cause a larger wound channel. Against stiffer targets (deer, bear, etc.), every bit of penetration is necessary to better the (poor) odds of a lethal shot with these (marginal) calibers.

6 Concluding remarks

The use of momentum as a lethality proxy in rifle calibers agrees with anecdotal evidence and “common sense”; small calibers are less lethal than large calibers, even though the small calibers travel much faster. Momentum does not directly address the issue of accuracy, although in general a caliber with a higher momentum will be less susceptible to wind. Large, flat bullets (e.g. .44 Remington MAgnum, .45-70 Gov’t, etc.) tend to lose velocity, and hence momentum, rapidly, making them highly lethal short-range cartridges. Pointed bullets (e.g. .375 H&H Magnum, .308 Winchester) retain more momentum at a given range, due to the lower drag of the bullet. Hence, these cartridges retain lethality close to the muzzle value at long ranges, making them suitable for long-range hunting.

For handgun calibers against human-like targets, the use of full metal jacket rounds may not be the most lethal, and hence the momentum-lethality relation breaks down at the higher momentums. For law enforcement officers, who are most likely to rely on a handgun for protection, this needs to be accounted for in the choice of handgun and ammunition.

In all cases, the use of momentum as a proxy for lethality assumes on near-perfect shot placement. Since shot placement is the single most important factor, the ability to accurately shoot a given caliber is more important than the momentum or theoretical lethality. For this reason, most law enforcement officers (and private citizens) are better equipped with a 9mm Luger handgun which they can accurately shoot, than a .45 ACP handgun with which they miss. Those who can accurately shoot the larger, faster calibers (.40 S&W, .45 ACP) will have an edge in lethality, which could make a difference in a hostage or defense scenario.